5. The Eviolin in Chamber Music

Timbre, in the general sense of varied instrumental combinations, is the primary organizing parameter in the chamber piece \textit{COALS}. In the first movement, timbre defines large-scale structure while a serialized family of pitch/rhythm motifs defines middle-scale structure. In the second movement timbre is serialized (though in a sense closer to Xenakis than to Schoenberg) together with pitch material and dynamics. This total-serial organization is applied only at the smallest scale, imposing short sequences on a large number of units each only several seconds long. These sequences gradually merge to produce a global structure true to the base materials. \textit{COALS} distils many concepts from chapter one (Boulez’s generalized serialism, Xenakis’s stochastics, Babbitt’s algebraic structures, Wolpe’s freedom) into a single artistic whole, using a synthetic instrument made possible from the heuristics of instrument design outlined in chapter two and the simplicial interpolator presented in chapter three.

5.1 Compositional criteria

\textit{COALS} is a large-scale composition for eviolin and five orchestral instruments. It demonstrates the eviolin and methodically explores its possible relationships with other instruments. A large universe of possibilities is severely constrained to shrink it to a manageable size, then thoroughly mined. This approach, maximum variety from minimum materials, owes a debt to Babbitt (or perhaps to the Vulcan proverb, “infinite diversity in infinite combinations”). Deliberate manipulation of structural parameters in the compositional process produces interesting patterns of growth and contrast, ones which my instinct alone would have been insufficiently imaginative to discover. A side effect of the methodical exploration of sub-ensembles in \textit{COALS} is that the piece feels light on its feet, without long tutti sections (a brief ten-note tutti occurs in bar 488, for example). This increases the perceptual density of information; paradoxically so, since the same work with some of these longer rests filled in would actually contain more data, but be at greater peril of being perceived as “everyone’s playing all the time.” More formally, this constant flux of sub-ensembles stays fresh even over a long time span by avoiding predictability.

The “broken consort” sound is inevitable with a newly invented instrument, unless we have the funds to build a whole family of such instruments. So \textit{COALS} uses a heterogeneous ensemble. Having accepted heterogeneity, the other instruments may as well come from different families. A representative combination of families, monophonic and polyphonic, percussive and sustained (and both: plucked and bowed strings), is given by flute, clarinet, eviolin, violoncello, trombone, and electric bass guitar. Since some
instruments in this ensemble are based on conventional equal-tempered semitones, and since all produce harmonic spectra, the sound synthesizer of the eviolin therefore also produces pitched sounds with harmonic spectra so the eviolin can work as a unit with the other instruments.

The physical gestures of the eviolinist are fairly quickly understood by listeners in how they affect the sound produced, so the score needs no specifically didactic elements which introduce one parameter at a time in clearly isolated contexts.

In terms of the five sonic dimensions postulated by Boulez—pitch, duration, amplitude, timbre, spatial position—the eviolin is not as different from the other instruments in the ensemble as one might anticipate. This comes from wanting not to make COALS a concerto for eviolin, but to arrange the instruments as structural equals. Certainly the eviolin attracts attention in performance because of its novelty and the spatial motion of its performer, but this need not be emphasized. The electronic synthesis of the eviolin could let it play far louder or quieter, more extremely and more subtly than the other instruments; recall the discussion of the aesthetics of listener saturation in chapter one. But this is not needed; if anything, after considering this aesthetic of “more” I find that it leads even more inexorably to the end of fertile lands than total serialism did. (But despite all the structures which enforce égalité and fraternité, liberté often favors the eviolin for solo roles.)

In particular, the eviolin’s pitch range, duration of short and long notes, and dynamic range are all similar to those of the other instruments. In each of these aspects, fineness of resolution is also conventional. Spatial position is fixed, for better blending with the ensemble; if only one instrument could cast its sound around the hall, solo/tutti opposition would be inevitable. But in timbre the eviolin outstrips its nonsynthetic companions. The range of spectral richness or thinness, brightness or dullness, produces a wider range of distinguishable timbres than is possible from the other instruments. From these stipulations, the structures which can be set up in each of Boulez’s five sonic dimensions will therefore be similar for all the instruments. Only in timbre does the eviolin go farther; in particular, it has two independent controls of timbre. This allows for quasi-modular structure: a gesture along spectral brightness (say, dull to bright and then back to medium) can be repeated with different values of spectral richness. This is of course not strictly modular in the sense of subdividing a continuum, but does produce structures which are heard similarly to pitch motifs repeated at different octave registers.
5.2 Structural analysis of COALS

COALS is in two movements. The first one, Camera obscura, is about 7 minutes long. The second, Carta terrae incognitae, takes 15 minutes. For brevity we often refer to the movements simply as Camera and Carta. (It is easy to remember which is first, from their alphabetical order.) The movements share some formal structures; certainly they share the same instrumentation. We first consider the things they have in common and then go on to particularities.

Inevitably this analysis is incomplete, almost exclusively describing the systematic aspects. The aspects which are imaginative (indisciplined, to use Boulez’s word again) are better heard in the music than read in an analysis.

5.2.1 Timbre

The gamut of raw timbral resources is mostly continuous, not discretely divided. Pizzicato and harmonics are the only discontinuous aspects of timbre, but even these are made to be continuous at a few points. In bar 115 of Carta, the cello bow smoothly picks up a decaying pizzicato note. Pitches repeated or continued with various spellings of natural harmonics occur regularly in the eviolin part: bar 61 of Camera, and bars 168, 354 and 525–528 of Carta. (The cello also does such in Carta, bar 352.)

A lattice of modules of timbre, the multidimensional timbre offered by a collection of diverse instruments, was not adopted for COALS. Though intriguing at first, the lattice’s considerable structure seemed unwarranted. Instead continuous and discontinuous paths are traced through a timbre space built up of instrumental combinations, detailed below in connection with figure 43.

As a point of departure for grouping instruments, COALS uses the simple model provided by Babbitt’s Composition for Four Instruments, namely using each combination exactly once. But within this egalitarian model still more criteria for grouping apply. Pairings exist within this ensemble, for example bowed strings, woodwinds, plucked strings, electrically amplified instruments. But a chain of pairs is not possible as with the ensemble of Le marteau sans maître. Instead we generalize the linear chain to a more interesting topology, one of relative distance determined by tessitura, frettedness, ability to play several pitches at once, and so on (figure 37). (This is literally a topology: in figure 37, \{\{f, c\}, \{t\}, \{e, v\}, \{e, b\}, \{e\}\} is a basis for a topological space whose open sets (unions of these basis elements) are

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157 Recall these pairings from chapter one: contralto voice and alto flute share human breath, alto flute and bowed viola monody, plucked viola and guitar the plucked-string timbre, guitar and vibraphone long resonance, damped vibraphone and xylophone pitched mallet timbre.
the sub-ensembles whose members share some of the attributes listed in the figure.) So depending on the view one takes, this heterogeneous ensemble has both unity and variety of timbre.

Figure 37. Various groupings of the instruments used in COALS, arranged to suggest relative distance between instruments. Instruments are abbreviated: (f)lute, (c)larinet, (e)violin, (v)ioloncello, (t)rombone, (b)ass guitar.

These pairings induce a coarse measure of timbral distance between instruments seen in the planar distribution in figure 37. These pairwise distances between instruments are then used to specify distances between sets of instruments. In Camera this set-distance emphasizes sub-ensembles with strong internal similarity (e.g., flute, clarinet, and trombone) or strong internal contrast (e.g., cello and flute). We will see that in Carta set-distance exhaustively defines a timbre space (figure 43), where the emphasized sub-ensembles are the ones more timbrally distant from others (e.g., flute and bass guitar at lower left: flute, clarinet, and eviolin at bottom center).

5.2.1.1 Timbre in Camera obscura

A germinal idea for Camera obscura was a traversal of all subsets of a set of size 4, i.e., an ordering or serialization. This ordering is listed in figure 38a. Successive elements of this ordering differ by the
addition or removal of exactly one element (exactly one digit changes between one and zero), making it what engineers call a Gray code. This particular code has two more properties which suit it to musical interpretation. First, it has a temporal asymmetry: compared to most Gray codes on four elements it has a wide spectrum, \textit{i.e.}, a variety of lengths of presence and absence of elements (the length of a run of ones or zeros in figure 38a). Second, it is spatially approximately symmetric in that each of the four elements has about the same number of presence/absence transitions. This can be visually verified: no single axis predominates in figure 38b.

An elaborated version of the code in figure 38, a Gray code on six elements, is the starting point for defining the succession of macroscopic timbres in \textit{Camera obscura}, namely the $2^6-1 = 63$ nonempty subensembles of six instruments. The literal succession defined by this code is split into five parts. The actual succession used to construct the score comes from shuffling the parts to form a pleasing overall shape. The boundaries between these five parts are carefully chosen to cause large discontinuities when the parts are alternatively juxtaposed as a result of the shuffling. Discontinuity here means the sudden entry or disappearance of considerably more than one instrument, such as the transition from solo clarinet to full ensemble at measure 131.

![Figure 38](image)

(a) A special Gray code on 4 elements. (b) Graphical derivation of this code. Vertices linked by an edge have three digits in common. A one in the first, second, third or fourth place corresponds to a point on the outside, top, front, or left half respectively.
In *Camera* the duration of each instrumental sub-ensemble comes from the internal similarities and dissimilarities of that sub-ensemble. In figure 37 we saw the instrumental groupings underlying the distances between instruments. Figure 39 extends this to a coarse measure of distance between instruments: pairs of instruments are considered similar, neutral, or contrasting depending on how many attributes are shared or opposed in the pair.

![Diagram](image)

(a) (b) (c)

Figure 39. Relative distance between instruments, based on the pairings shown in figure 37. Lines indicate (a) similarity, (b) neutrality, and (c) contrast. Instruments are abbreviated as in figure 37: (f)lute, (c)larinet, (e)violin, (v)ioloncello, (t)rombone, (b)ass guitar.

Within a sub-ensemble, then, several pairwise comparisons are possible. For brevity we here consider only the case of three instruments, in which three pairwise comparisons occur; similar patterns hold for sub-ensembles of other sizes. Figure 40 shows how sub-ensembles with strong similarities and/or contrasts receive longer durations. These durations are related exponentially, not linearly, since this is how we perceive duration (recall the critique in chapter one of Xenakis’s vector spaces). These raw duration-ratios then slowly vary on the scale of the whole movement, to create a variety of harmonic rhythms.
Figure 40. Relative duration as a function of the number of similar, neutral, and contrasting pairs of instruments in a sub-ensemble of size 3. (Since three pairs exist in such an ensemble, the top three numbers in each column sum to three.)

5.2.1.2 Pitch, gesture, and rhythm in *Camera obscura*

We have seen the structural outline of *Camera obscura* at the level of timbre. Within this there are gestural and pitch structures. Figure 41a lists the eight primitive gestures from which the movement is built up.
These structures of pitch and gesture are tightly correlated. Each of the gestures $a$ through $h$ is rendered exclusively with its corresponding pitch class set in figure 41b. For example, at the very beginning of the movement flute and clarinet clearly state gesture $a$ with pitch classes (in order) B, E, F, D, Bb. Since these gestures have considerable freedom of interpretation in the musical surface—dynamics, assignment to instruments, durations, and so on—this is not as restrictive as it first seems. Some of Beethoven’s motivic repetition is far more constrained than this, for instance. The sequential order of pitch classes in each gesture is free as well, as long as the relative size of intervals in the gesture is respected.

At bar 59 eviolin and cello introduce a derived collection of pitch class sets, stating gesture $a$ on the pitches Bb D Eb E A. This derived collection comes from remapping individual pitch classes: the whole-tone scale on C remains unchanged, but the whole-tone scale on C# is transposed up four semitones. This results in pitch sets which have a recognizable overlap with the originals, yet also have different internal structure because semitone intervals become four- or five-semitone intervals (figure 41c). This new pitch
mapping remains in force until a solo clarinet passage builds up into a *tutti* return at bar 131 of the original pitch mapping.

Figure 42a restates figure 41b to emphasize the common elements among these pitch sets. To produce a slow circulation through the chromatic aggregate, the gestures/pitch sets are serialized in cyclic fashion as shown in figure 42b: *abe*, *bcg*, *cfh*, *bfg*, *bdeg*, *beg* go completely around a circle in clockwise direction.

![Diagram](image)

As this circulation proceeds, the most common gestures/pitch sets are *b* and *g*: looking within pitch sets, the most common pitch classes are B, D, and Eb. These tend to be de-emphasized in the musical surface by appropriate choice of dynamics, register, and instrumentation in order to increase the overall information density of the movement. On the other hand, the less common gestures/pitch sets *a*, *d*, and *h*, together with pitch classes C, C#, A, and G, are generally reserved for rendering more dramatic structural events. This correlation of rare things with rare things actually decreases information density but clarifies
structure: several new things occurring simultaneously are more easily recognized as a structural articulation.

The circulation is sometimes reversed and sometimes initiated at a different place, corresponding in traditional serialism to retrogression and Stravinskian rotation respectively. The speed of circulation also changes as the movement unfolds, in order to vary the harmonic/gestural rhythm.

In summary, there are correlations between pitch, rhythm, and gestural content in *Camera obscura*. These are analogous to the correlations between timbre, dynamics, and pitch content in *Carta* (figure 43), to which we now turn.

![Figure 43. The space of timbres in *Carta terrae incognitae*, the distance between all subsets of instruments. Instruments are abbreviated as in figure 37: (f)lute, (c)larinet, (e)violin, (v)ioloncello, (t)rombone, (b)ass guitar.](image)

5.2.1.3 Timbral definition of structure in *Carta terrae incognitae*

There are $2^6 - 1 = 63$ points in the space of timbres shown in figure 43, namely each nonempty subset of the whole ensemble. The six instruments are not equidistant in figure 37, so the space is naturally
represented by a 6-cube distorted by these unequal distances. The constraint imposed on the space represented in figure 43, then, can be imagined as a crinkled planar slice or shadow of this 6-cube. This crinkling produces the irregular mesh in figure 43; it further distorts the distances, but reduces the number of edges in the graph. Though the planar graph suggests a triangulation of this set of points, it is not complete in this sense: the thinned-out triangulation is also an effort to reduce the number of edges. The reason behind this, in turn, is to keep the structures built on this graph down to a manageable size.

The edge set of this graph is partitioned with directed paths. The paths are constructed iteratively by depth-first traversal of the edges. Each path begins at a vertex chosen uniformly randomly; whenever a path reaches a vertex with more than one edge leaving it, an edge is chosen also uniformly randomly. Each edge is used only once. When a path gets stuck because no more edges leave the vertex it has reached, it is called complete and a new path is begun. This process continues until all edges have been used.

This partitioning is actually done twice. This creates redundancy in the paths and in the musical surface creates recognizable repetitions of timbral successions, rather like slow chord progressions. These repetitions are not very long, though, since for any given edge of the graph the two paths containing it, one from each partitioning, will diverge from each other not far before or after that edge. The second partitioning also adds a constraint to the depth-first traversal: path lengths are bounded. This produces more shorter paths than the first partitioning and thereby increases variety.

The paths are arranged in small sequences to imaginatively balance conjunct and disjunct motion in all three parameters, timbre, dynamics set, and pitch set. A concatenation of a few long paths is continuous in timbre and discontinuous in dynamics and pitch, while a concatenation of many short paths is discontinuous in timbre (at the boundaries between paths) but can be continuous in dynamics and/or pitch. An example of the former is found in the slow orchestral change but rapid harmonic rhythm of bars 445–467. The latter is exemplified by bars 285–298, an almost Mahlerian fluctuation of instrumental forces supporting slowly evolving harmony and a gradual curve from f to pp and back. Within each path, the duration of an edge simply corresponds to its length in figure 43.

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158 A formal definition of distance between sets of instruments, derived from the distances between individual instruments, seems too laborious when only six instruments are involved. Such a set is still tractable for manual computation. For larger sets the formal definition would likely include ideas such as the Levenshtein distance ("minimum editing distance") between two strings (Levenshtein 1966, Allison 1984). Mean pairwise distance, though simple, is inaccurate for cases like the distance between the set of all six elements and a subset with one element removed.

159 This distortion extends to a structural level the "local indiscipline" favored by Boulez. His words bear repeating: "at the overall level there is discipline and control, at the local level there is an element of indiscipline—a freedom to choose, to decide and to reject" (Boulez 1976, 66).
The temporal model has one other level: these small sequences are themselves sequenced imaginatively to create the overall form. The largest-scale form thus gradually anneals or emerges from the low-level material itself, not from an arbitrary external model. It is a cooperation between formal structure and imagination. Babbitt’s question of what makes something appropriate to map into a musical parameter is easy to answer here: large-scale ordering comes not from a skyline or coastline but from the innards of what has been already composed. No charge of inappropriateness can even be laid.

Global intensity contours are then overlaid on this largest-scale sequence. In the musical surface, overall intensity is rendered as intensities of individual instruments. Intensity and dynamics are entirely independent parameters: an intense pp and a low-intensity ff are just as possible as a high-intensity ff. Flute and clarinet intensity is defined simply as register, since these instruments produce such different timbres—warm, thin, penetrating—when lower or higher notes are played. Among its other controls, the eviolin’s intensity control is forwards/backwards position, which drives spectral brightness: the backward extreme is nearly sinusoidal, and the forward extreme approximates a buzzy square wave. Cello intensity is rendered as sul tasto / sul ponticello; pizzicato is sometimes used as an extreme case of sul pont. playing. The trombone uses both a plunger mute and bell position; these could be used independently but COALS uses them as a unit. Low to high intensity therefore ranges from muted through modo ordinario to bell-up unmuted. The electric bass guitar uses a pair of pickups for intensity control: low to high intensity involves selecting the bass pickup, both bass and treble, or treble only. The player does this by operating switches or knobs on the guitar body. No other effects units are used with the guitar, though its volume knob sometimes sustains or increases amplitude during a note.

The rendering of a global intensity contour as a collection of particular intensities of individual instruments is comparable to how a pitch set or dynamics set is rendered as a set of simultaneous pitches or dynamics of individual instruments. These three rendering techniques, intensity, pitch, and dynamics, share yet another property: sometimes one instrument takes on several values at the same time, for example when more values need rendering than there are instruments playing. As pitch this is commonly seen as a trill; in dynamics an instrument jumps between several levels in a phrase or sequence of little separated neumes; in intensity, jumping between levels is again the case.

In summary, these formal structures agrees with the spirit of total serialism in that they serialize many musical parameters together with pitch. But the structures here are richer—and deliberately more perceptible—than the literal mappings of Structures Ia.
5.2.2 Large-scale temporal organization

Motives, gestures, and phrases vary in size in COALS. Some are only a bar long, others are longer. There is a spectrum of motive-length and phrase-length, like the range of phrase lengths in the music of Mozart. But unlike the music of Mozart, phrases in COALS do not form hierarchies. A better word may be parallelarchy. In everyday life this idea is seen more often spatially than temporally, for instance weather maps which indicate the variations of precipitation, temperature, barometric pressure, and so on over some region of the landscape, that is, a spatial rather than temporal extent. These properties correlate to various degrees. Where all these properties simultaneously take on extreme values or rapidly change in value, we apply dramatic names like “cold front” or “tornado.” But this is the exception; both in weather patterns and in COALS, particularly at smaller levels, phrases tend to overlap rather than line up. Not everybody arrives at once.

At the time scale of a minute or so, sections are elided rather than simply concatenated. These joints between sections are heard differently by different listeners, depending on which attributes they attend to more in figure 43: timbre, dynamic content, or pitch material (or intensity, though this is a little more abstract than the first three). Each rule for sectional division suggests a particular reading of the musical structure, as shown for an excerpt in figure 44.

Figure 44. Different parsings of sectional division in Carta terrae incognitae, bars 108–180.

COALS fluidly moves between polyphony and heterophony instead of building up nested aggregations, the polyphonies of heterophonies of polyphonies suggested in (Boulez 1963a, 119–141). In particular, one class of heterophonies which becomes more and more prominent from bar 314 to the end is the duet. In
this context a duet is any grouping of two instruments, possibly with other instruments playing simultaneously. For example, in bars 314–317 cello and bass guitar engage in a loose exchange of quiet mutterings; the cello then pairs with eviolin to bar 323 in a lyrical ascent while the bass protests in brief outbursts. As the movement progresses, the two parts of a duet become less independent. In the very last duets such as the unaccompanied clarinet and eviolin in bars 535–540, heterophony even begins to allude to unison playing.

5.2.3 Tempo and duration

Pulse occasionally sets up a regular, striated time amenable to acceleration/deceleration. Particularly *Carta* has occasional pulse trains for a dozen or so beats. For example, bars 92 to 94 thereof decelerate through a running eighth note passage in eviolin. (This gradual deceleration and sudden *a tempo* are strengthened by exact correlation with amount of flute vibrato.) In a related passage in bars 125–128, a “walking bass” line suddenly accelerates in mid-stride. Where tempo changes occur, both gradual and sudden (or momentary as with a fermata), the exact amount of change is left indeterminate. But for the majority of both movements the musical surface has no pulse, creating irregular, amorphous time with no possibility of acceleration or deceleration but only a variable density of events. No simultaneous multiple tempos therefore need occur; a single conductor beating a single pattern suffices to synchronize the players.

Highly flexible use of grace notes makes pulse by and large inaudible despite the quarter-note tactus and simple time signatures often evident on the page. This is particularly the case with longer runs of grace notes, which by the standards of common practice cannot fit into the time provided. It is up to each performer to decide where to steal the time from, which synchronizations to make with the lines of the other performers and which to abandon.

Like some long compositions by Morton Feldman, *COALS* loses a sense of forwards and backwards motion. In the latter case at least, this is because the underlying formalisms are symmetric with respect to the direction of time. But unlike Feldman’s predilection for small and very large time scales, the structures in *COALS* operate primarily with moderate durations on the order of 5 to 90 seconds.

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160 The depth-first traversal of the graph in figure 43 is, strictly speaking, asymmetric with respect to time. But at this abstract level if a five-minute block of *Carta* were recomposed with paths reversed, the block could hardly be identified without prior knowledge. Even as a purely graph-theoretic exercise forward and backward paths would be almost indistinguishable.
The performers of *COALS* do not have to deal with many explicitly notated indeterminacies. It occasionally happens, as with the viola in bars 151–152 where floor position is not specified directly but rather indicated to vary quickly through its whole range, or as with instructions to hold a note until out of breath. Implicitly, however, both tempo and dynamics are less determinate than one might expect. Within the four common dynamic markings of *pp, p, f*, and *ff* lies plenty of latitude for the performers to make their mark; we now discuss the origin of this dynamics system.

### 5.2.4 Dynamics in *Carta terrae incognitae*

Dynamics are more often point indications than crescendos or decrescendos. As the graph in figure 43 is traversed, it induces a sequence of dynamics sets (analogous to pitch sets). At each point of the graph, dynamics are chosen only from the set labeling that point’s horizontal slab, one of the fifteen nonempty subsets of \{pp, p, f, ff\}. The boundaries between horizontal divisions in figure 43 tend towards equal numbers of points in each horizontal slab, but deliberately leave some inequality for dramatic effect. Total “democracy,” a uniform distribution, would transmit the greatest amount of raw information to the listener. But more interesting are occasional reductions of information rate, so for example the unique occurrence of *tutti pp* in the second row from the bottom in figure 43 stands out in the overall dramatic flow, much more so than the half-dozen occurrences of the dynamics sets in the middle rows.

The fifteen dynamics sets are distributed among the rows to approximate uniform distributions of two readily audible properties of a dynamic set, the spread from quietest to loudest dynamic and the average dynamic. So scanning down the left column of figure 43 there is no discernible trend from, for example, louder to quieter or broad range to narrow range. In other words, position in figure 43 is strongly decorrelated with dynamic level and dynamic range. This allows for the perceptibly distinguishable construction of sections with deliberately continuous dynamic properties, by appropriately sequencing many short segments. Such continuity is unlikely to happen accidentally when the decorrelation is strong.

We have seen how the rows of figure 43 are deliberately chosen to contain different numbers of points. Thus, the dynamics sets are also distributed to strongly decorrelate (in average level and in spread) with the number of points in each row. Without this decorrelation some kinds of dynamics set (loud ones or narrow ones) would predominate at the expense of their opposites. For a composition as long as *COALS*, avoiding such an unequal distribution better maintains long-term interest. This is because a uniform distribution maximizes the rate of information flow at the global time scale on which such a distribution is perceived.
Since the sequence of dynamics sets comes from a traversal of the graph in figure 43, the onset of a new dynamics set can only occur when a new subset of instruments begins (a new point in the graph is reached). About half the time this means that a new instrument is introduced. So in rendering dynamics sets, if a new dynamic is also introduced, it is preferably assigned to the new instrument so that both structures, dynamics and timbre, are heard more clearly.

5.2.5 Pitch organization in *Carta terrae incognitae*

The several basic harmonic areas in *Carta* label the columns of figure 43. These harmonic areas we will generally call pitch sets (understood to mean pitch class sets). As with dynamics sets, the eventual order of these pitch sets in the musical surface is determined by the traversals of the graph in figure 43 and the subsequent multilayered sequencing of the paths produced.

Within each occurrence of a pitch set, the limited availability of pitch classes suggested that (as with tonal or pentatonic music) they be organized in a hierarchy. In some cases one pitch class is specified as central, and perhaps another as a “dominant” to this “tonic”. In other cases, particularly five-element pitch sets, the pitch classes are organized with a structure developed for this movement which I call a trichord series. This structure is explained in detail below.

5.2.5.1 Content of pitch sets

I wanted pitch structure to reflect timbral structure, but I could not use the distance between pitch classes directly as with the distance between timbres. This is because the distance between pitch classes is difficult to quantify pragmatically. By comparing simple frequency, two pitch classes a semitone apart are nearby while two a perfect fifth apart are almost maximally distant. But the exact opposite holds if we compare interval consonance (call it overlap of overtone structure, to safely retreat from aesthetics to objective psychoacoustics). In a large composition it is quite possible for each metric to hold at different times. Since any resolution of these opposing metrics would be complicated, I followed the conclusions of Vriend (1981) from studying elaborate techniques of Xenakis and instead chose a much simpler construction.

Twelve points were placed uniformly and randomly in a square; each point was labeled with a pitch class. Drawing the Delaunay triangulation of these points (for only twelve points, doing this by hand is straightforward) produced a graph of points connected by edges, analogous to figure 43 but having points
corresponding to pitch classes instead of timbres. So the same depth-first traversal applied as before: choose a point, choose an edge leaving that point (excluding previously chosen edges) to reach another point, and keep on going until you get stuck; repeat this until you run out of edges. As it stands this traversal would produce an ordering of pitches, but instead I needed a collection of sets of pitches to apply to the columns of figure 43. So I constrained the path-construction algorithm above, similarly to how the second traversal of the graph in figure 43 was constrained: terminate the path early if five points have been visited. This produced pitch sets of size at most five; smaller ones resulted from paths which got stuck sooner.

The resulting pitch sets are listed across the top of figure 43. The assignment of pitch sets to columns derives from criteria much like that of dynamics: uniform distribution of readily audible properties such as spread and size. Pitch sets have no average value, only a spread. This simplifies the assignment: pitch set average need not be decorrelated with the number of points in each column.

To maintain interest over the global duration of this movement, just as with dynamics sets the size of pitch sets is decorrelated with the number of points in each column. For example, of the two smallest pitch sets \{Bb, C#\} occurs only 3 times while \{Eb, A\} occurs 13 times. The five-element sets occur from 4 to 10 times each. So no size consistently predominates at the expense of another; this maximizes the information rate of this attribute. As with the horizontal boundaries between dynamics sets, the vertical boundaries tend towards equal numbers of points in each column but deliberately leave some inequality for dramatic effect.

5.2.5.2 Expanded pitch sets

The basic pitch sets listed in figure 43 are sometimes stretched, like the music of Brahms and his successors stretches harmony but preserve a skeleton sometimes more imagined than evident in the musical surface.

Boulez-like frequency multiplication is the simplest technique used in Carta to expand a pitch set. In bars 175–212 (with interruptions at bars 190–198 and 204–205) figure 43 would dictate that only the pitch set \{G#, A, C\} be used. Instead, this pitch set is expanded into its whole-tone flat and sharp neighbors,

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161 When drawn most compactly as in figure 43, the spread of a pitch set is defined as the interval between the lowest and highest pitch. Strictly speaking, according to this definition a pitch set does have an average value, the mean between these two extrema. But this is not nearly as perceptible as the average value of a dynamics set, under normal circumstances when more than one octave is used. Pitch set average is therefore correspondingly less important to decorrelate with other parameters as a means of increasing information density.
i.e., multiplied by the pitch set \{F\#, A\#\}. This expansion is furthermore constrained so that choice of neighbor is correlated with dynamic level. Instruments playing \textit{pp} use \{F\#, G\#, A\#\}; \textit{ff}, \{A\#, B, D\}; \textit{f} or \textit{p}, the original pitch set.

A similar correlation is found in bars 108–121, where the pitch set \{D, Eb, E, G, Ab\} is multiplied by \{C\#, Eb\}. The effect here is dramatic because the pitch set shifts by semitones all at once in a block presentation. This is a result of the concentration of dynamics sets with only one element in this passage. At the \textit{tutti pp} in bar 111, the pitch set shifts up a semitone. Three bars later a \textit{tutti ff} pushes it down two semitones to one below normal. Unequal dynamics return at bar 116, where \textit{pp} and \textit{p} are rendered one semitone sharp, \textit{ff} and \textit{f} one semitone flat; this correlation is reversed two bars later. Finally in bars 120–121 a \textit{tutti ff} occurs, but now all dynamics revert to the original pitch set.

The pitch set \{G\#, A, C\} occurs again in bars 160–172. Here \textit{pp} and \textit{p} instruments go up a semitone, \textit{f} and \textit{ff} down a semitone. A variety of dynamics combinations—all quiet, all loud, some of each—themselves produce quite a varied emphasis of intervals.

In bars 273–276 and 301–305 \{G\#, A, C\} is multiplied by a tritone interval to make \{G\#, A, C, D, Eb, F\#\}. These passages are too short to benefit perceptibly from correlating pitch extension with other parameters.

Frequency multiplication may be too dignified a term for modifications such as the one which occurs in bars 283–285. Here eviolin and trombone briefly nudge the pitch set \{G\#, A, C\} up a semitone, simply for variety. This brief burst of what is ostensibly \{G\#, A, C\} occurs in the middle of a sea of \{Ab, Bb, B, C, C\#\}, so its break with orthodoxy is not so easily noticed. (This may be what Ligeti was thinking of in applying serial techniques to error itself: an error too mild to be consciously noticed during a performance, as contrasted with more serious deviations from the formal system underlying a composition.)

Because of the narrow compositional possibilities and common occurrence of the pitch set \{Eb, A\} as it stands in figure 43, it is sometimes compressed in time (its predecessor and successor pitch sets invade its frontiers). This pitch set is also sometimes expanded to larger related pitch sets rather like the expansion just mentioned in bars 108–121. Figure 45 illustrates perhaps the most extreme modification. To the original set \{Eb, A\}, its semitone neighbors \{D, E, Ab, Bb\} are added to get \{D, Eb, E, Ab, A, Bb\}. From this set, \{Eb\} is actually dropped to produce a pitch set of size five, which lets it be structured as a retrograde trichord series on \{E, A, Bb, D, Ab\}.
5.2.5.3 Trichord series

Trichord series are structures in *Carta* which explicitly group different members of a pitch set and thereby generate small-scale harmonic motion as well as intervallic variety. A single trichord series is a Babbittesque sequence of all ten ways to choose three elements from a set of five, in this case all trichord subsets of a pentachord. Arranging these trichords in a sequence, operations analogous to those of dodecaphony apply. The prime form comes from the canonical ordering of choices, shown abstractly in the top row of figure 46 and in an especially transparent musical context in figure 47.
Figure 46. Trichord series on a set of five pitch classes. The four staves show the prime, retrograde, inverse, and inverse retrograde forms respectively.

The retrograde form simply reverses the order of the ten trichords (figure 46, second row). For example, bars 567–570 contain a retrograde series on {D, F#, Eb, G, F}. Another retrograde series in bars 266–269 soon after the excerpt in figure 47 is on {G, F#, F, Eb, D}, the same pitch classes of the preceding series but in a different order. This strongly resembles the series in figure 47, since the retrograde series of a reversed pitch set has the same first two and last two trichords as the original. The inverse form of the series starts with the outermost two elements of the prime form and jumps back and forth until the middle is reached (figure 46, third row). Like its dodecaphonic namesake, the inverse retrograde form is both the retrograded inversion and the inverted retrogression; that is, it starts at the middle and oscillates outwards (figure 46, fourth row). Since the noninverted forms of the series have a more static harmony, more common pitch classes between successive trichords, the inverted forms are more common in COALS.

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162 This is a slight abuse of mathematical notation. The braces connote pitch set, but the set is ordered. Many different series can come from different orderings of the set. Orderings in COALS are determined on a case-by-case basis from the surrounding musical context. Unlike the presentation in figure 47, they often avoid consecutive elements a semitone apart in order to increase intervallic variety.
Trichord series can be rendered flexibly, just as with dodecaphonic series. Here are some examples.

An inverted series is compressed in bars 501–503 (figure 48). Bars 285–286 contain the first nine trichords of a series; the completing tenth chord is delayed until the downbeat of bar 291, analogous to the delayed completion of a chromatic aggregate in dodecaphony. At sufficient speed, the harmonic function of a trichord series gives way to melodic functions. This is the case in bars 446–448 where cello and trombone present an unusually fast inverted series on \{D, Eb, F, F#, G\} (figure 49).
Figure 48. *Carta terrae incognitae*, bars 501–503. An inverted trichord series compressed by eliding notes common to successive trichords. Trichords are circled in the diagram.

The series is built on the pitch classes \{Bb, C, Ab, C#, B\}.

Figure 49. *Carta terrae incognitae*, bars 446–448. An unusually fast inverted trichord series on \{D, Eb, F, F#, G\}.

In bars 372–386 a succession of solos in flute, trombone, bass guitar and eviolin each render a trichord series in arpeggiated fashion. The same series, the prime form on \{C#, B, Ab, C, Bb\}, is used in all cases so these solos could even be called quasi-canonic imitation. Only one series is used because arpeggiation would obscure the series order of an inverted form beyond recognition. Pitch class counters should beware of the glissandi in the trombone part, as a glissando from C# down to B includes the pitch class C.

A concentration of trichord series is found at the very end of *Carta*. Bars 552–563 (with a short interruption) present an inverse retrograde followed by an inverted series on \{Ab, Eb, G, D, E\}. After a longer interruption, bars 567–573 conclude the movement with retrograde and prime series on \{D, F#, Eb, G, F\}.
5.2.5.4 Regions of static harmony

As with much of the music of Wolpe, the serial yet nondodecaphonic treatment of pitch in *COALS* avoids aggregate grayness. Subsets of the chromatic aggregate circulate sometimes quickly, sometimes slowly. In fact, when the longest edges of figure 43 are traversed the pitch material goes beyond coherence and approaches stasis.

The pitch set \{B, C\#, E, F\} is the one most frequently associated with long durations. Fortuitously it contains all interval classes from semitone to tritone (an “all-interval tetrachord”) so intervallic variety is maximized within this tight constraint. Even so, a few chromatic inflections like appoggiaturas are sometimes found in these passages. This is the case in the flute/eviolin/bass guitar trio in bars 122–128 and the flute and eviolin duet in bars 247–251. On the other hand, bars 190–194 strictly adhere to the pure pitch set; moreover, long chords without vibrato bring out the stasis. Emphasized stasis is desirable in this passage because it is both preceded and followed by a few dozen bars of highly active playing. (The immediately preceding bars 175–189 actually correspond to the very longest edge in figure 43, but avoid harmonic stasis by subjecting the pitch set \{G\#, A, C\} to frequency multiplication as we have seen.) Closer to the end of the movement, bars 453–458 are again fairly pure, adding only four whole-tone (not semitone) inflections and a glissando. Stasis is brought out here by long unchanging trills and tremolos in the eviolin part.

The very first harmonically static section occurs in bars 18–31 on \{Eb, A\}, one of the two smallest pitch sets. A few chromatic inflections occur here, such as in bar 23 where the clarinet introduces a trumpet-like gesture which returns several times in the movement.

Finally on the pitch set \{G\#, A\#, B, C, C\#\}, there are two long duets for clarinet and eviolin in bars 122–127 and near the end of the movement in bars 535–540. Since this is one of the largest pitch sets, maintaining it for an extended period reduces harmonic motion less obviously than is the case with the smaller pitch sets.

5.2.6 Motives and gestures

Short gestures are common in both movements of *COALS*: in the first by specification as we have seen, in the second by a more intuitive recognition of similar patterns being susceptible to similar motivic treatment. These little motives recur sometimes soon, sometimes much later, sometimes literally, sometimes only as an allusion not immediately recognizable. Such short gestures fit into the formal constraints.
more easily than long melodies. They also allow for rapid changes of texture, enabling a fluidity which is desirable when large-scale organization comes from something other than gradual Lisztian development of a few principal themes. Where Liszt orchestrates to deliver a thematic narrative, COALS if anything does the reverse by using motives to support and connect orchestral (timbral), dynamic, and pitch structures. As a perhaps unflattering analogy, Humphrey Bogart movies use physical action to help deliver a gripping story whereas 1990’s action films like *Independence Day* and *Rumble in the Bronx* use narrative plot merely as an excuse to display heroic feats. Another medium provides a more favorable comparison: painters after Liszt’s day began to work nonrepresentationally (i.e., nonnarratively), leading eventually to the formal definitions of *De Stijl*, a group which influenced Boulez as we saw in chapter one. But even so, specialists can argue which was means and which was end centuries earlier, Vermeer’s portrait or Vermeer’s use of light and shadow.

Even though motives do not play a primary structural role in *Carta*, we can consider a few examples of their recurrence in different contexts. Though separated by great distances, two easily recognizable repetitions (because of the little trumpet-call introduction) are the flute line in bars 464–467 and the clarinet in bars 146–150, which spring from the clarinet passage starting at bar 23. Even the pitch classes are preserved in bars 146–150. The immediate continuation of the clarinet line, bars 151–155, returns in the bass guitar at bars 299–302. (This continuation is also partially begun by the flute in the example just mentioned, after bar 467). Another gesture, “quasi glissando” semitone sliding, occurs early on in the clarinet, bars 44–45; it recurs in the clarinet as late as bars 536–538.

Multi-instrumental ideas also recur. The complex interplay between flute and clarinet in bars 277–280 is restated by only the flute as a compound line in bars 293–295, followed immediately in the next four bars by an amplified restatement using both flute and clarinet again. These restatements differ markedly from the original because they project it into only two pitches. The longest motivic recurrence in *Carta* is naturally enough a loose imitation and elaboration: the duet between cello and trombone in bars 129–138 is found again in the context of a different pitch set in the flute and clarinet, several minutes later in bars 273–282. In both passages the prominent duet is quietly accompanied by two other instruments.

The most expanded motivic idea occurs a little past the middle of *Carta*. In bars 321–332 the trombone pitches trace out a three-level fractal pattern, schematized in figure 50a. This is followed immediately by an inverted four-level pattern in the cello, bars 333–355 (continuing on another six bars in free imitation of this texture), figure 50b. (The bass guitar comments on the cello line, emphasizing certain notes, tracing
out its own little fractal lines at times.)\textsuperscript{163} Despite the length of these fractal processes, monotony is hardly a danger because so much else changes while the process unfolds: pitch set, dynamics, intensity, rhythmic cells, note groupings (never threes within threes), internal reflections within the fractal in the second case, and speed of presentation.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fractal_melodies.png}
\caption{Fractal “melodies” in \textit{Carta terrae incognitae}. (a) Schematic of the pitch levels in the trombone part, bars 321–332. (b) Schematic of the pitch levels in cello, bars 333–355. (c) The last third of (b) leading into free elaboration, as it appears in bars 349–355.}
\end{figure}

From late Beethoven (and many after him) \textit{COALS} borrows the technique of long-range linear connections in pitch. For example, in bar 26 of \textit{Camera obscura} the eviolin reaches a high F while other instruments play in less elevated tessitura. Four bars later the flute connects it to a high E, where the

\textsuperscript{163}The parameter of register is already occupied in flute and clarinet with rendering the amount of intensity, but it is available in these lower three instruments.
connection is left hanging for a while. The eviolin takes it up again briefly with a B flat in bar 44, and only in bar 49 does the flute finally carry this line down to connect to the rest of the musical surface. Such connections occur even more frequently with low notes, since low notes have a fixed playable limit while high ones vary with performer skill. Again in *Camera obscura*, the trombone briefly visits its bottom octave and bottoms out on a G in bar 103; this register is entirely avoided until the bass guitar suddenly continues the motion downward to a single G flat in bar 122, then four bars later exactly retraces the steps of the cello back up, remembered from over a minute ago.

5.2.7 Problems of notation

Most of the notation in *COALS* is prescriptive: do this. But this is seasoned with occasional descriptive notations: make it sound like this. Examples of the latter include imprecise Italian adjectives and fermatas. (This conscious decision to combine them follows Brian Ferneyhough’s example, as mentioned in chapter one.)

No extraordinary notations are used to simplify learning and rehearsal. Performers might ignore a few parameters in early rehearsals, like a choir may ignore dynamics or replace text with a neutral syllable on the first reading of a piece. The relatively ventilated texture even makes playing from full score possible since pages can be turned during rests. Playing from parts would necessitate more than the average number of cue lines, since a synchronizing pulse is for the most part absent but subtle timing relationships between instruments are common. Another practical consideration is the generous amount of vertical white space in the score. This leaves room for occasional dense accumulations of “diacritical” markings: *sul pont.*, dynamics, slurs, left-hand pizzicato markings, mute instructions, and so on.

Rhythmic notation is deliberately simple, to counter the sometimes high complexity of everything else. At least synchronization in rehearsals is made easier without challenges like overlapping, out-of-phase, or nested tuplets; multiple simultaneous tempos; or sudden jumps between precise metronome markings. Escape from the tyranny of the pulse comes through an (in some ways) easier method, grace notes. These produce deliberate local desynchronizations and occur in any of four configurations, following a note or rest and preceding a note or rest. Each grace note sounds distinctly despite its shortness: their purpose is desynchronization, not ornamentation or being secondary in any way.

One innovation in rhythmic notation in *COALS* is the accenting of rests. An accented rest tells the performer to strongly mark the end of the preceding note. The note should not die out, but its ending should be
felt as a rhythmic event in its own right. (My first organ teacher made an impression on me by pointing out that, unlike piano playing, the end of a note is just as important as the beginning.)

One extended technique used in *COALS* which has no widely accepted notation is a sequence of diminishing vertical bars on the staff, to indicate the bouncing of a bow on the strings without appreciable transverse motion.¹⁶⁴ This is seen for example in *Carta* bar 43, cello and bar 73, eviolin. In other cases, English phrases (footnotes) are often clearer than special notation for infrequent events. For example, for the flute overblowing at bars 9 and 11 of *Carta* a graphical depiction of the higher harmonics encountered would have been challenging to typeset and somewhat perplexing to read. The instructions to play until out of breath or out of bow in *Camera* bars 94 and 134, or how to align a grace note figure shared by flute and clarinet in *Camera* bar 116, are more efficiently communicated through prose than through clever graphic symbols. Only if one of these events occurred numerous times would a special graphical notation be more efficient for the performer to decode.

### 5.2.7.1 Graphical notation of spatial movement

Notating the horizontal position of the eviolin took a few tries to get it right. Strictly speaking, for eviolin this technique is not extended; but as violinists outnumber eviolinists we practically consider it to be so anyways. An early score separately graphed each parameter versus time (figure 51a). This failed to clearly communicate latitude and longitude because those are not just one but two coupled spatial parameters, and two parameters cannot fit onto one vertical axis. They need to be coupled on the score as well, to correspond to the performer’s mental conception of moving around: heading for a spot on the stage, not turning two knobs on an Etch-a-Sketch. The multi-graph layout is useful for considered analysis but poor for real-time performance. Again in hindsight, a four-staff layout is difficult because human performers do not have four independently focusable eyes. Score-reading remains a multidimensional tracking task as we have seen. Fewer separate areas to look at keep it a single task instead of a multitude of unidimensional tracking tasks. In the latter case, the score cannot be read so much as memorized, one parameter at a time if necessary.

¹⁶⁴ This notation, apparently originated by Benjamin Britten, was brought to my attention by David Bohn.
Figure 51. (a) Separate graphs of each parameter versus time. (b) Dot-in-square notation, including direction arrows. (c) Dot-in-square notation as used in COALS (*Camera obscura*, bars 131–134, eviolin).

So with reduced spatial resolution but increased clarity and concision, a later study patterned indications of spatial position after indications of dynamics (figure 51b): small glyphs inserted at points in the score. These square glyphs were either *subito* or contained arrows analogous to *p crescendo* to indicate future direction of movement. These arrows turned out to be more cluttering than helpful so they were replaced in COALS with dotted lines connecting squares, analogous to a hairpin marking between *p* and *f*. This final notation is shown in figure 51c.